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SHAPE OPTIMUM DESIGN OF TRUSSES UNDER MULTIPLE LOADING

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Abstract—Shape optimum design of elastic trusses under multiple loading conditions is considered. The optimization procedure includes selection of topology, geometry and sectional properties. The weight of a truss is minimized subject to nodal equilibrium and permissible stress constraints, and constraints to ensure a unique stress-free length of each member. The approach presented here helps to reduce the nonlinearity of constraints. The problems of realizability and overstress in the final design, as present in some of the previous works, are easily met without any conceptual difficulty.

NOTATION

A^p	cross-sectional area of the pth member
E	Young's modulus
F^i	ith component of external force vector
k	counter for Cartesian coordinate direction, takes values 1, 2 and 3
L^p	length of the pth member at the equilibrium state
L_i^p	stress-free length of the pth member
l^{ip}	direction cosine between the position vector represented by the pth member, with the origin at the node where F^i acts, and a base vector in the direction of F^i
m	total number of members
n	total number of degrees of freedom
ni	number of members meeting at the node where F^i acts
p	count for truss members
q	count for the loading conditions
r	total number of loading conditions
T^p	axial force in the pth member
T^{ip}	axial force in the pth member connected to the node where F^i acts
W	objective function
y_k	kth component of Cartesian coordinates defined at each equilibrium state
Δy_k^p	difference of y_k between the two ends of the pth member
Δy^{ipq}	difference between the y_k-coordinates of one end of the pth member and the other end which is connected to the node where F^i acts, k being the same component of Cartesian coordinates with the direction of F^i under the qth loading
Δy^{ip0}	Δy^{ipq} at the loading-free state
ρ^p	density of the pth member
ε^p	axial strain in the pth member
δ^p	minimum specified cross-sectional area of the pth member
σ_t	permissible stress in tension
σ_c	permissible stress in compression
$(\ )^p$	value for the pth member
$(\ )^q$	value for the qth loading state
$\sum_{pi}^{ni}$	summation for all ni members meeting at the node where F^i acts.

1. INTRODUCTION

Shape optimization in discrete structures introduces several degrees of additional complexity compared to optimization on fixed shapes. This is primarily due to an increased number of decision variables, an increased nonlinearity of these variables and the potential change in topology during optimization.

The literature on optimization with variable geometry is rather scarce. Most of the methods available in the literature for shape optimization (Vanderplaats and Moses, 1972; Lev, 1977) require an optimization process and a separate structural analysis to be carried out for checking a design point. This makes the optimization process complex and costly, even with the present decreasing costs of numerical computation (Saka, 1980). Saka (1980) presented a conceptually simple and direct approach employing the global stiffness equations as constraints, without the need for a separate structural analysis to check a design point. Typical truss structures were designed to demonstrate that shape optimization yields results which are better than those derived by methods not using nodal coordinates as design variables.

Another concept that is recognized for improving the minimum weight design is that of prestressing due to lack of fit at the loading-free state (Hofmeister and Felton, 1970; Felton and Dobbs, 1977). Spillers and Levy (1984) have shown that for design under two loading conditions the optimum design with prestress is fully stressed. This is, however, not so for the design without prestress.

Optimum design under two loading conditions was studied by Spillers and Lev (1971) and Spillers and Levy (1984) who used the so-called sum and difference approach. Even for design with a fixed geometry, the sum and difference approach has several problems. The problems lie in the fact that the sum and difference approach for trusses always results in two solutions with different topology for the sum and the difference loading conditions. These are then overlaid by adding the required cross-sectional areas for each loading to obtain a solution. The solution so obtained may cause overstress in the resulting design, even though each of the individual designs satisfy the permissible stress constraints (Spillers and Lev, 1971). A more severe problem encountered in the application of this method, as discussed by Spillers and Levy (1984), is that it is not possible to realize the truss.

When the sum and difference approach is extended to r loading cases, the number of separate minimum weight problems to be analyzed increases by the $(r-1)$th power of two (Lev, 1977). This has algebraic difficulties (Lev, 1977), in addition to the problems of realizability and overstress already mentioned.

It is well recognized that both prestressing at the loading-free state and shape optimization lead to an improved minimum weight design. There is no work that makes use of both these approaches together for design under multiple loading conditions.

The present work is an effort to make a contribution to shape optimum design of trusses under multiple loading conditions. The weight of a truss is minimized subject to nodal equilibrium and permissible stress constraints, and constraints to ensure uniqueness of the stress-free length of each member. The continuity of displacement at nodes is automatically satisfied by treating coordinates defined at each of the loaded equilibrium states as independent variables. Hence, continuity conditions of displacement are excluded from the constraints. The approach presented here, as compared to that of Saka (1980), helps to reduce the nonlinearity of constraints. The problems of realizability and overstress in the final design under multiple loading conditions are easily met without any conceptual difficulty. The method is general in that it can consider any number of loading conditions on the truss. Further, there are no uncertain assumptions made in the analysis. Two numerical examples are included to demonstrate the proposed formulation.

2. FORMULATION OF DESIGN PROBLEM

The aim of this study is to investigate the shape optimum design of elastic trusses under multiple loading conditions. It is assumed that, based on past experience, an initial topology and geometry can be selected. The optimization procedure includes selection of topology (presence or absence of members and joints), geometry (location of joints in the coordinate space) and sectional properties (cross-sectional areas). In this study, only those members and joints which are initially supplied to define a truss are retained or deleted. New members or joints are not added to the original topology.

In the final design, some of the members may have zero cross-sections. As a check against instability of the final design, members crucial to overall stability of the truss need to have their lower bounds for cross-sectional properties specified to be greater than zero.

In order to simplify the analysis it is assumed that (1) materials are linearly elastic, (2) strains are small compared to unity, (3) truss members are prismatic between nodes, (4) all loads are conservative, and (5) self-weight of each truss member is considered as lumped concentrated weight at both of its ends.

2.1. *Shape optimum design under single loading condition*

Selecting an appropriate measure W as the optimality criterion, the design problem is formulated as follows:

$$\text{minimize } W \tag{1}$$

subject to nodal equilibrium constraints, stress constraints and minimum requirements of cross-sectional areas, expressed as

$$F^{iq} - \sum_{pi}^{ni} l^{ipq} T^{ipq} = 0 \tag{2}$$

$$l^{ipq} - \frac{\Delta y^{ipq}}{L^{pq}} = 0; \quad L^{pq} - \left[\sum_{k=1}^{3} (\Delta y_k^{pq})^2 \right]^{1/2} = 0 \tag{3,4}$$

$$\sigma_t A^p - T^{pq} \geqslant 0, \quad \text{when } T^{pq} \geqslant 0 \tag{5a}$$

$$\sigma_c A^p + T^{pq} \geqslant 0, \quad \text{when } T^{pq} < 0 \tag{5b}$$

$$A^p - \delta^p \geqslant 0 \tag{6}$$

where (y_1, y_2, y_3) = Cartesian coordinate system defined at each of the loaded equilibrium states; $(\ )^q$ = value for the qth loading, which may include the loading-free state as well; $(\ )^p$ = value for the pth member; F^i = ith component of external force vector in one of the Cartesian coordinate directions; Δy^{ipq} = difference between the y_k-coordinate of one end of the pth member and the other end which is connected to the node where F^i acts, k being the same component of Cartesian coordinates with the direction of F^i under the qth loading; T^{pq} and T^{ipq} = axial force in the pth member of a truss and that connected to the node where F^i acts, respectively, under the qth loading; A^p = cross-sectional area of the pth member; σ_t and σ_c = permissible values specified for tensile and compressive stresses, respectively; ni = number of members meeting at the node where F^i acts; $\sum_{pi}^{ni}$ = summation for all ni members meeting at the node where F^i acts; δ^p = minimum specified cross-sectional area of the pth member; L^p = length of the pth member at the equilibrium state; and l^{ip} = direction cosine between the position vector represented by the pth member, with the origin at the node where F^i acts, and a base vector in the direction of F^i. The ranges of the parameters are (1) i = 1–n, (2) p = 1–m, and (3) q = 1–r, where n = total number of degrees of freedom; m = total number of members; and r = total number of loading conditions which is equal to one for the case of single loading condition.

From the view point of structural analysis, it is noted that the formulation includes only equilibrium equations at the nodes but does not include the member stiffness equations and the continuity conditions of displacement at the nodes. It is also noted that, by the definition of coordinates at the loaded equilibrium states, equilibrium is considered at each of the displaced configurations. The continuity of displacement is automatically satisfied by treating coordinates defined at each of the displaced equilibrium states as independent variables.

The tensile permissible stress is constant for a given material. The compressive permissible stress is generally a function of the stability parameter. For a practical design, it is to be selected from appropriate formulae given in design specifications. Since the stability parameter for a member depends on the length and the cross-sectional area, eqn (5b) is a nonlinear constraint.

If the weight of a truss, W, is taken as the objective function, it is expressed, with due consideration of the assumption of small strains, as

$$W = \sum_{p=1}^{m} \rho^p A^p L^p \tag{7}$$

where ρ^p = material density of the pth member.

There is no constraint to ensure that the stress-free lengths of members are compatible when assembled. Hence, in the design of indeterminate trusses, the proposed formulation generally results in a solution with the presence of prestress due to lack of fit at the loading free state. The magnitude of stress in each member due to this prestress can be checked by performing structural analysis calculating back from the loaded state. There is, however, the possibility that this stress exceeds the permissible stress. If it is preferable to have a design in which this stress always remains within the permissible stress constraints or even at a lower range for ease of construction, the loading-free state has to be considered as one of the loading states resulting in a design under two loading conditions.

If design without prestress is preferable for an indeterminate truss, an additional constraint that the stress-free length of each member is equal to the length between the nodes connecting that member at the loading-free state is necessary. The stress-free length of the member with length L^{pq} under the action of internal force T^{pq} can be obtained by solving the following three equations

$$\sigma^{pq} = \frac{T^{pq}}{A^p}, \quad \sigma^{pq} = E\varepsilon^{pq}, \quad \varepsilon^{pq} = \frac{(L^{pq} - L_i^{pq})}{L_i^{pq}} \quad (p = 1, 2, \ldots, m;\ q = 1, 2, \ldots, r) \tag{8–10}$$

as

$$L_i^{pq} = \frac{L^{pq}}{1 + \dfrac{T^{pq}}{A^p E}} \tag{11}$$

where ε = axial strain; E = Young's modulus; and L_i^{pq} = stress-free length of the pth member calculated back from the qth loading equilibrium state. It is noted that eqn (9) is the elastic constitutive relation and eqn (10) is equivalent to the strain–displacement relation. Introducing the coordinates of nodes at the loading-free state as additional variables and incorporating them with eqn (11), the constraint is expressed with $q = 1$ as

$$L^{p1} - \left(1 + \frac{T^{p1}}{A^p E}\right)\left[\sum_{k=1}^{3} (\Delta y_k^{p0})^2\right]^{1/2} = 0 \tag{12}$$

where $\Delta y_k^{p0} = \Delta y_k^{pq}$ at the loading-free state. Equation (11) can be used to obtain the stress-free length of each member. Then, L^p of eqn (7) can be replaced with L_i^{p1} of eqn (11), though the resulting difference is of no significance numerically.

2.2. *Shape optimum design under multiple loading conditions*

For design under multiple loading conditions, the equilibrium and permissible stress constraints of eqns (2) through (5) have to be considered for each loading case. An additional constraint has to be considered for a unique stress-free length of each member when calculated back from the state under each of the different loading conditions. For the case of design under r multiple loading conditions, the additional $(r-1)$ constraints to ensure a unique stress-free length for each member are given, in view of eqn (11), as

$$\frac{L^{p1}}{1+\frac{T^{p1}}{A^{p}E}}-\frac{L^{pq}}{1+\frac{T^{pq}}{A^{p}E}}=0 \quad (p=1,2,\ldots,m;\ q=2,3,\ldots,r). \tag{13}$$

Similar to the case of single loading, the proposed formulation generally results in a solution with the presence of prestress at the loading-free state in the design of indeterminate trusses. If design without prestress is preferable, the same additional constraint as expressed by eqn (12) is to be imposed.

It is essential for a practical design that the final shape of a truss be acceptable from engineering and architectural viewpoints. Thus, additional constraints as listed below can also be considered: (1) some joints of a truss have to be fixed to resist external forces; (2) a set of joints may be located symmetrically about an axis; (3) some joints may be restricted to move within a confined area; and (4) a set of joints of a truss may be restricted to move along fixed lines.

3. COMPARISON OF THE PROPOSED FORMULATION WITH THOSE IN THE LITERATURE

There is no literature on shape optimization, including both a change of geometry and topology, under multiple loading conditions with prestress at the loading-free state. Most of the methods proposed for minimum weight design under multiple loading conditions consider only the variation of the topology of the trusses (Hofmeister and Felton, 1970; Spillers and Lev, 1971; Sheu and Schmit, 1972; Reinschmidt and Russell, 1974; Lev, 1977; Felton and Dobbs, 1977; Spillers and Levy, 1984).

One of the methods proposed in the literature for minimum weight design under two loading conditions with a fixed geometry (Spillers and Lev, 1971; Lev, 1977; Spillers and Levy, 1984) is the so-called sum and difference approach. In the sum and difference approach for two loading conditions, the minimum weight design is obtained using the plastic analysis method by analyzing the minimum weight designs for two single loading cases $(F^1+F^2)/2$, the sum solution, and $(F^1-F^2)/2$, the difference solution, where F^1 and F^2 represent the two loading conditions. The optimum member areas are obtained by adding the area requirements for each of these two single loading conditions. The plastic analysis method is used since it makes the trusses determinate in the analysis, thus making the computation easier. The optimum design using the plastic analysis method is not due to the intention to evaluate it based on plastic behavior but just a tool to obtain the optimum design based on elastic behavior. It is, however, not at all clear whether a truss so designed can be an optimum design based on elastic behavior (Spillers and Lev, 1971). The problem lies in the fact that the sum and difference solutions for trusses based on the plastic analysis method always result in a topology that is statically determinate even when the original topology is indeterminate. Since the determinate trusses so obtained are not identical in topology, the truss which results from the overlay of these two topologically different determinate solutions is usually indeterminate. The load redistribution resulting from the combination of these two single loading designs may cause overstress in the resulting design, even though each of the individual designs satisfies the permissible stress constraints under the plastic analysis. Spillers and Lev (1971) have studied this problem and concluded that the sum and difference design for two loading conditions is conceptually difficult.

Lev (1977) has extended the sum and difference approach to more than two loading conditions for design of determinate trusses, starting from a topology which is usually indeterminate. Use is made of the decomposition method proposed by Spillers (1972) to convert the minimum weight design problem under r independent loading conditions into $(2)^{r-1}$ minimum weight problems under the sum and difference loads. An algorithm is presented to remove the redundant members and arrive at a determinate optimum solution. Such a consideration leads to an increased problem size as the number of separate minimum weight problems to be analyzed increases by the $(r-1)$th power of two. The use of Lev's algorithm (Lev, 1977) to remove the redundant members to achieve a determinate optimum solution has considerable algebraic difficulties for problems of such a large size. A similar consideration for indeterminate trusses would, as discussed above, lead to the problem of overstress in the final design. A more severe problem encountered in the application of this method, as discussed by Spillers and Levy (1984), is that it is not possible to realize the truss. There is no literature on the application of the sum and difference method to minimum weight design for a variable geometry under multiple loading conditions.

After having concluded in an earlier work (Spillers and Lev, 1971) that the sum and difference design for two loading conditions is conceptually difficult as reviewed earlier, Lev (1977) and Spillers and Levy (1984) have used it again in two separate later works. This may be taken to imply that a better approach for design under multiple loading conditions was not available.

An approach for shape optimum design is to temporarily ignore some of the conditions among the equilibrium equations, member stiffness equations and continuity conditions of displacement. Usually, member stiffness equations are ignored in the optimization process. After a number of iterations or after arriving at an optimum design, depending on the algorithm used, the stiffness equations for the whole truss are assembled and an exact structural analysis is carried out. This method has been applied for minimum weight design both for a fixed (Reinschmidt and Russell, 1974) and variable geometry (Vanderplaats and Moses, 1972).

Reinschmidt and Russell (1974) used equilibrium and permissible stress constraints to determine the optimum topology and member cross-sections of a truss for a fixed geometry under multiple loading conditions. If the optimum design so obtained is indeterminate, an exact structural analysis utilizing the global stiffness equations is carried out separately. The values of the internal forces so computed are used to redesign the member cross-sections and the optimization process is repeated until the convergence criterion is met. The separate analysis utilizing the global stiffness equations was considered as not being necessary for the design of determinate trusses and hence excluded. This is not true for design under multiple loading conditions, because the solution does not result in a unique length of each member at the loading-free state. Thus when multiple loading conditions are considered, a separate structural analysis has to be carried out not only for indeterminate trusses, as suggested by Reinschmidt and Russell (1974), but for determinate trusses as well. As the global stiffness equations are used only to revise the cross-sectional areas and are not included as constraints in the optimization problem, the final design is not necessarily the minimum weight design. On the other hand, the resulting design will be fully stressed since cross-sectional areas are always revised to minimum values necessary to satisfy the permissible stress constraints.

Vanderplaats and Moses (1972) proposed an algorithm for minimum weight design of a truss with a variable geometry. The optimization is carried out first for a given fixed geometry of a truss, and then the joints are moved in coordinate space to obtain an improved minimum weight design. The process is repeated until a satisfactory convergence is attained. Equilibrium and continuity conditions of displacement are imposed in the fixed geometry optimization stage of the algorithm. The fixed geometry design was achieved using the familiar stress-ratio algorithm which seeks a fully stressed design. It has been shown by Schmit (1960) and Kicher (1966), however, that the optimum fixed geometry design may not be obtained for indeterminate trusses using the stress-ratio algorithm.

The methods of Vanderplaats and Moses (1972) and Reinschmidt and Russell (1974) require a separate structural analysis for checking a design point. This makes the entire optimization process computationally expensive, though the process might have been necessary for the optimization to converge and to obtain a solution at that time. Even small trusses with only a few design variables may have a large number of equations for structural analysis to be solved during the design process (Saka, 1980). Saka (1980), then, presented a conceptually simple and direct approach using the global stiffness equations together with a limitation on displacement and stress, including the effect of buckling, as the constraints for the optimization problem. The member areas, nodal coordinates and joint displacement were considered as design variables. Multiple loading conditions can be considered without any of the associated problems mentioned earlier. When design under prestress is to be carried out, the approach of Saka (1980) needs to consider additional variables for the lack of fit of the component members in a highly nonlinear system of global stiffness equations.

The method proposed in this study for shape optimum design is general in that it can consider any number of loading conditions. In addition to the equilibrium constraints, the constraint of eqn (13) is considered to ensure that the stress-free length of each member is unique when calculated back from the state under each of the multiple loading conditions. As obvious from the derivation, these constraints are equivalent to the elastic constitutive relations and strain–displacement relations. With the constraint of eqn (13), the problem of realizing the so optimized trusses under multiple loading conditions is easily overcome for the stress free length. The proposed method does not have the problem of overstress in the final design, as was encountered in some of the past works (Spillers and Lev, 1971; Spillers and Levy, 1984), since design is made for all the loading conditions simultaneously rather than an overlay of topologically different determinate solutions. For each additional loading considered after the first loading condition, only one additional constraint is introduced beyond the equilibrium equations.

The formulation of this study has at least two definite advantages over the work of Saka (1980) and may have an additional advantage as well. One of the two definite advantages is the exclusion of the continuity conditions of displacement from the constraints of the proposed formulation. The necessity of imposing continuity conditions leads to the necessity of introducing member stiffness equations as constraints. These continuity conditions and member stiffness equations are included in the global stiffness equations used in the formulation of Saka (1980). The second advantage is in the design of prestressed trusses where the proposed formulation does not require the constraint of eqn (12). This makes the formulation for a prestressed design at the loading-free state simpler than that without prestress. However, the formulation by Saka (1980), for design with prestressing, requires additional variables to take into account the lack of fit in the expression of the already complex global stiffness equations. This second advantage is a significant improvement of this study compared to the previous works, which becomes clearer for the case of single loading. The elastic constitutive relation and strain displacement relation for each member, which when combined are equal to the member stiffness equation, are not included in the formulation.

By the first advantage, the optimization process of the proposed formulation for the design of trusses free of prestress at the loading free state requires less computational effort than that of Saka (1980). By the second advantage, the optimization process of the proposed formulation for the design of prestressed trusses at the loading-free state requires less computational effort than the design of trusses free of prestress, while that of Saka (1980) requires more effort. Since both advantages contribute, the design of prestressed trusses at the loading-free state is much more efficient when compared with the formulation of Saka (1980).

Another possible advantage is in the optimization algorithm. With complex nonlinear constraints in the form of global stiffness equations, as formulated by Saka (1980), there is no way to take advantage of the knowledge of structural characteristics in the optimization algorithm and the use of a general nonlinear optimization algorithm is inevitable. Though no study has been made, it is felt that, since each constraint in the proposed formulation is simple and the physical meaning of each constraint is obvious, the presentation of the proposed formulation may lead to a better optimization algorithm. Such an algorithm could take advantage of the knowledge of the structural relevance of each constraint in the optimization problem. A study of these aspects is one of the topics for a future study.

In order to ensure that the member forces due to prestressing do not exceed the permissible stress for the design with prestress at the loading-free state, both the proposed formulation and that due to Saka (1980) have to include the initial loading-free state as one of the multiple loading states. It may even be preferable to limit the internal axial forces within a certain range for ease of erection. This can be attained by additional constraints specifying the limits on internal axial forces under the loading-free state.

The formulation of Saka (1980) is based on the small displacement theory, while, as pointed out earlier, the proposed formulation is based on the finite displacement theory. The consideration of finite displacement, however, does not introduce any additional

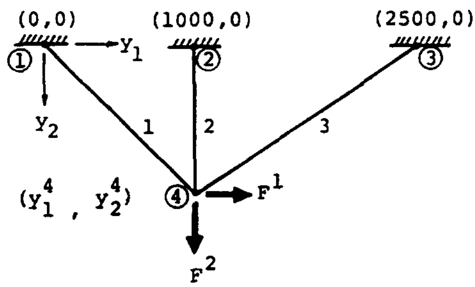

Fig. 1. 3-Bar truss under multiple loading (Example 1).

numerical complexity. Rather, it results in the exclusion of continuity conditions of displacement from the constraints. Except for this difference of constraints, the difference in these two approaches may not be significant, but at least conceptually the finite displacement theory is considered to be more exact than the small displacement theory.

The objective function selected is the weight of the truss. However, it can also be taken as the cost of construction reflecting both the material and fabrication costs, if necessary, though the objective function will then vary for each fabrication plant.

4. DESIGN EXAMPLES

The proposed method is demonstrated in two design examples. The sequential quadratic programming method (Gill *et al.*, 1986) is used to get numerical solutions. In the figures for the following examples, the bare numbers, the numbers in the circles and the numbers in the parentheses indicate truss elements, nodes and coordinates of nodes, respectively.

Example 1

The 3-bar truss, shown in Fig. 1, is taken as the first example to demonstrate the advantage of shape optimization with prestressing for a minimum weight design under two loading conditions. The cross-sectional areas and forces of the three members, and the y_2-coordinate at node 4 are taken as variables in this design problem. A starting solution with $y_2^4 = 1000.0$ mm was employed. For a direct comparison of the design with and without prestressing at the loading-free state, the same permissible stress of 0.14 kN mm^{-2} is used both in tension and in compression. The modulus of elasticity is specified as 207 kN mm^{-2}. A lower limit of 10 mm^2 is placed on the cross-sectional area of all members and 500.0 mm on the y_2-coordinate at node 4. Loading cases 1 and 2 are specified as $\langle F^1 \;\; F^2 \rangle = \langle 700.0$ kN $\;500.0$ kN$\rangle$ and $\langle 500.0$ kN $\;700.0$ kN$\rangle$, respectively. In changing from a prestressed design to the one without prestressing, the additional constraint of eqn (12) is to be considered. The prestressed design is 20% lighter than the design without prestress. It is fully stressed while that without prestress is not. The final results are given in Table 1.

Table 1. Optimum design for Example 1

	Design with prestress		Design without prestress	
Variable	Loading 1	Loading 2	Loading 1	Loading 2
T^{11}	790.53	580.00	575.43	431.40
T^{21}	141.37	436.95	304.78	541.32
T^{31}	−1.40	−0.90	−180.63	−96.44
y_1^4	500.00		500.00	
y_2^4	980.33		1020.00	
A^1	5646.67		4110.23	
A^2	3121.06		4092.07	
A^3	10.00		1966.64	
Volume	7.792×10^6 mm^3		9.789×10^6 mm^3	

Coordinates in mm; member forces in kN; cross-sectional area in mm^2.

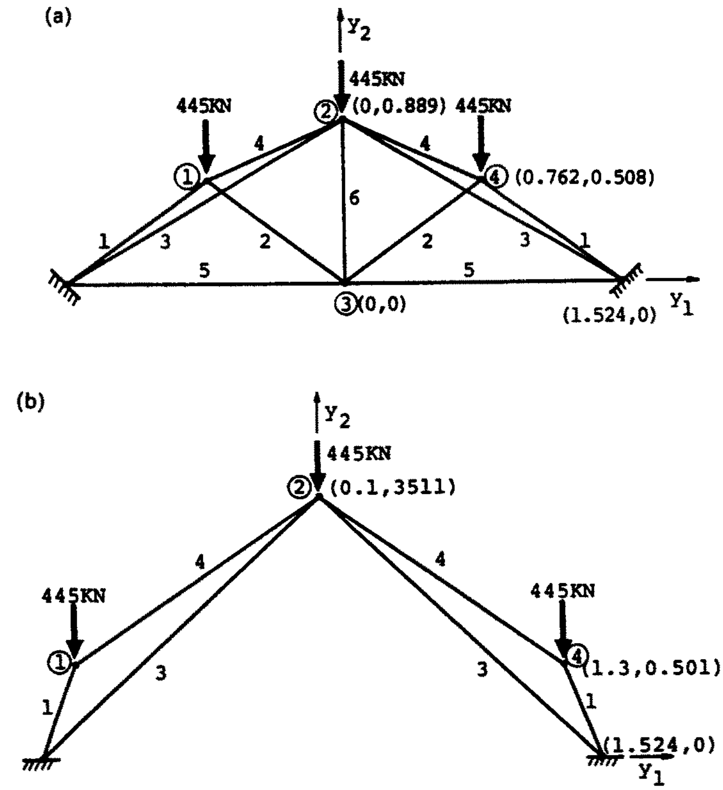

Fig. 2(a). 11-Bar symmetrical truss (Example 2). (b). Optimum design by the proposed method (Example 2).

Example 2

The 11-bar indeterminate symmetrical truss subjected to vertical loads, shown in Fig. 2(a), is taken from the example of Saka (1980). Considering the symmetry, six member areas, three coordinates (y_2^2, y_1^4, y_2^4) and six member forces are treated as variables in this design problem. The permissible compressive stresses for groups of members 1, 3 and 4 are kept constant at 0.1378, 0.1163 and 0.1159 kN mm^{-2}, respectively, the same constant value as used by Saka (1980) regardless of the stability parameters, so that the results of the formulation of this study can be directly compared with that of Saka (1980). The allowable tensile stress is 0.149 kN mm^{-2} and the modulus of elasticity is 207 kN mm^{-2}. A lower limit of 500.0 mm^2 is specified on the cross-sectional area of member 3 to maintain the stability of the structure. The coordinate limits specified are 510.0 mm < y_1^2, 250.0 mm < y_1^4 < 1300.0 mm and 0.0 mm < y_2^4 < 1500. 0 mm.

The proposed method resulted in an optimum topology which is determinate and similar to that obtained by Saka (1980). It may be noted that, since the optimum solution is determinate, the truss can be assembled at the loading-free state without any prestress. The solution by the proposed method has a volume of 1.653×10^7 mm^3, which is less than 1.688×10^7 mm^3 obtained by Saka (1980). It is also interesting to note that the solution obtained by the proposed method is less than 1.689×10^7 mm^3, which is obtained for the case of a 6-bar truss with the top chord along the funicular polygon.

The optimum solution by the proposed method and that due to Saka (1980) are listed in Table 2. The optimum solution is shown in Fig. 2(b).

5. CONCLUSIONS

While optimum design with prestress at the loading-free state has been investigated for trusses under no shape change, there are few works in the literature for shape optimization, including both a change of topology and geometry, under multiple loading conditions. Except the work of Saka (1980), most of the methods available in the literature for design

Table 2(a). Nodal coordinates for Example 2

		Initial solution	Optimal solution (Saka)	Optimal solution (Proposed)
Nodes (cm)	y_2^2	88.90	131.25	135.11
	y_1^4	76.20	130.00	130.00
	y_2^4	50.80	51.00	50.09
Volume	(cm^3)	4.280×10^4	1.688×10^4	1.653×10^4

Table 2(b). Cross-sectional areas for Example 2

		Initial solution	Optimal solution (Proposed)
Member areas (cm^2)	A^1	70.00	50.00
	A^2	30.00	0.00
	A^3	40.00	5.00
	A^4	45.00	28.99
	A^5	0.10	0.00
	A^6	30.00	0.00

under multiple loading suffer from the difficulty of realizability and possible overstress in the final design. The method proposed in the present study has been able to overcome these problems and is simpler than that of Saka (1980). This is especially so when prestressing is preferred at the loading-free state. There are no uncertain assumptions made in the formulation.

The proposed formulation is based on the finite displacement theory while that of Saka (1980) is based on the small displacement theory. Due to the use of the finite displacement theory in the formulation, the continuity conditions of displacement at nodes are not necessary. These conditions are, however, included in the global stiffness equations used as constraints in the method of Saka (1980). Because of this, the proposed method requires less computational effort than those of Saka (1980) for the design of prestress. For trusses with prestress, the proposed formulation requires less computational effort than the design free of prestress, while that of Saka (1980) requires more computational effort. Because of these, the design of prestressed truss is much more efficient when compared with the method of Saka (1980). Besides, there exists a significant difference in the presentation of constraints in this formulation and that of Saka (1980). This difference of presentation of constraints may make some difference in the optimization algorithm.

As reported in the literature, the advantage of prestress in design under two loading conditions with shape optimization is confirmed in Example 1.

Although only weight has been considered as the objective function, additional criteria like the cost of fabrication and optimization of the prestressing forces or a combination of these can also be considered. This will be the subject of a future study.

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